CHAPTER 2 : FUNCTIONS AND GRAPHS

- Basics of functions
- Graphs of functions
- Transformations of functions
- Combinations of functions : Composite functions.
- Inverse functions.

2.3) Basics of functions

Relations

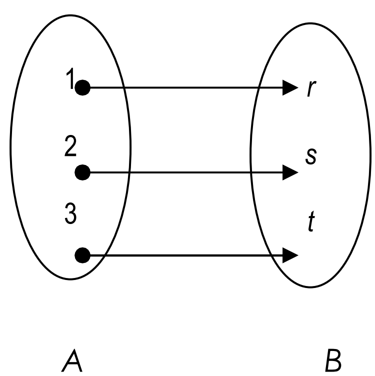
Any set of ordered pairs (a,b) where $a,b \in \Re$ is called a **relation**.

The set of all *a* is called the **domain** of a relation and the set of all *b* that is related to *a* is called the **range** of a relation.

For example :

The relation between set *A* and set *B* is given by {(1,r), (2,s), (3,t)} where *A*= {1,2,3} and *B* = {r,s,t}

We can represent the relation as shown in figure below



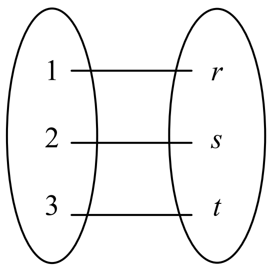
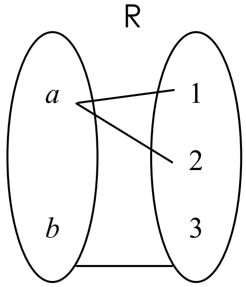
The domain of relation = {1, 2, 3}

The range of relation = {*r, s, t*}

Functions

A function is a correspondence (relation) of **each element** in the domain to **exactly one element** in the range.

Example :

Relation	Description
R 	R is one to one correspondence. Each element in the domain corresponds to only one element in the range. $\therefore$ R is **a function**
R 	R is one to many correspondence. Element *a* in the domain corresponds to element 1 and element 2 . $\therefore$ R is **not a function**
R = {(1,6), (2,6), (3,8), (4,9)}	R is a **function** because each element in the domain corresponds to exactly one element in the range.
R = {(1,2), (3,4), **(5,6) , (5,8)**}	R is **not a function** because element 5 in the domain corresponds to 2 elements in the range which are 6 and 8.

Example :

Determine whether an equation below represent a function .

(a) $x^2 + y = 4$ (b) $x^2 + y^2 = 4$ (c) $x + y^3 = 27$

solution :

(a) Given : $x^2 + y = 4$

Solve the equation for *y*

$$y = -x^2 + 4$$

For each value of x there is exactly one value for *y*.

Therefore , by definition, $x^2 + y = 4$ is a function.

(b) Given : $x^2 + y^2 = 4$

Solve for *y*

$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4 - x^2}$$

For each value of x there are 2 values for *y* because of $\pm$

By definition, $x^2 + y^2 = 4$ is not a function.

(c) Given : $x + y^3 = 27$

Solve for *y* :

$$y^3 = 27 - x$$

$$y = \sqrt{27 - x}$$

For each value of x there is exactly one value for *y*. By definition, $x + y^3 = 27$ is a function.

Function Notations

If *y* is a function of x , we may use the notation $y = f(x)$. The notation $f(x)$ is read as '*f* of x' where $f(x)$ is a value of *f* at x.

Example :

$f(2) = 5 \Rightarrow$ the value of *f* at 2 is 5

$f(-5) = 0 \Rightarrow$ the value of *f* at -5 is 0

Evaluating a function

Example :

If $f(x) = x^2 + 3x + 5$ then evaluate the following :

(a) *f*(2) (b) *f*(-5) (c) *f*(*a* + *h*)

(d) $f(r^2)$ (e) the value of *f* at 10

(f) find x if the value of *f* is 15.

(g) $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$

Solution :

Given : $f(x) = x^2 + 3x + 5$

(a) $f(2) = 2^2 + 3(2) + 5 = 4 + 6 + 5 = 15$

(b) $f(-5) = (-5)^2 + 3(-5) + 5 = 25 - 15 + 5 = 15$

(c) $f(a+h) = (a+h)^2 + 3(a+h) + 5 = a^2 + 2ah + h^2 + 3a + 3h + 5$

(d) $f(r^2) = (r^2)^2 + 3(r^2) + 5 = r^4 + 3r^2 + 5$

(e) The value of f at 10 $= f(10) = (10)^2 + 3(10) + 5 = 135$

(f) If $f(x) = 15$, what is x ?

$\Rightarrow x^2 + 3x + 5 = 15 \Rightarrow x^2 + 3x - 10 = 0 \Rightarrow (x+5)(x-2) = 0$

By applying zero - product principle :

$(x+5) = 0$ or $(x-2) = 0$

$x = -5$ or $x = 2$

(g) $f(x+h) = (x+h)^2 + 3(x+h) + 5 = x^2 + 2xh + h^2 + 3x + 3h + 5$

Therefore :

$$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 + 3x + 3h + 5 - (x^2 + 3x + 5)}{h}$$

$$= \frac{2xh + h^2 + 3h}{h} = \frac{h(2x+h+3)}{h}$$

$$= 2x + h + 3$$

<u>Exercise :</u>

(1) Given that $f(x) = x^2 - 2$ and $g(x) = 2x - 3$, find and simplify each of the following expressions.

(a) $f(-3)$ (b) $g(5)$ (c) $f(a+1)$ (d) $\dfrac{g(x+h) - g(x)}{h}$, $h \neq 0$

(2) Find each of the following for the function $f(x) = \sqrt{2x-1}$

(a) $f(5)$ (b) x , if $f(x) = 0$ (c) $f(a+2)$

Answer :

(1) (a) $f(-3) = (-3)^2 - 2 = 7$

(b) $g(5) = 2(5) - 3 = 7$

(c) $f(a+1) = (a+1)^2 - 2 = a^2 + 2a - 1$

(d) $g(x+h) = 2(x+h) - 3 = 2x + 2h - 3$

$$\frac{g(x+h) - g(x)}{h} = \frac{2x + 2h - 3 - (2x - 3)}{h} = \frac{2h}{h} = 2$$

(2) (a) $f(5) = \sqrt{2(5) - 1} = \sqrt{9} = 3$

(b) $f(x) = 0 \Rightarrow \sqrt{2x-1} = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = 1/2$

(c) $f(a+2) = \sqrt{2(a+2) - 1} = \sqrt{2a+3}$

Piecewise – defined Functions

For some functions different formulas are used in different regions of domain. Such functions are called **piecewise-defined function**.

Example :

$$f(x)=\begin{cases} x\ , \text{ for } x\geq 0 \\ -x, \text{ for } x<0 \end{cases}$$

for $x\geq 0$, a formula $f(x)=x$ is used to obtain a certain value and for $x<0$, a formula $f(x)=-x$ is used.

Example (Evaluating piecewise-defined function) :

If given $f(x)=\begin{cases} x^2+3, & x<0 \\ 5x+3, & x\geq 0 \end{cases}$, then find :

(i) $f(-5)$ (ii) $f(6)$ (iii) $f(0)$

Solution :

(i) Since $x=-5<0$, use the first formula $f(x)=x^2+3$

$$f(-5)=(-5)^2+3=25+3=28$$

(ii) Since $x=6>0$, use the second formula $f(x)=5x+3$

$$f(6)=5(6)+3=30+3=33$$

(iii) Use the second formula :

$$f(0)=5(0)+3=3$$

Exercise :

(1) If $g(x)=\begin{cases} 6x-1, & x<0 \\ 7x+3, & x\geq 0 \end{cases}$ then find $g(-3), g(0) \text{ and } g(4)$.

(2) Given :

$$f(x)=\begin{cases} x^2+2x+1, & x<-2 \\ x+1, & x\geq -2 \end{cases}$$

Find : (a) $f(-3)$ (b) $f(5)$

Domain of function

Tips on finding the domain of a function :

- If the given function has **neither division nor even root**, the domain of the function is all **real number** (R).
- If the given function has **division**, **exclude any value that make division by zero** in the domain.
- If the given function has **even root**, make sure the **radicand is positive number** to get the real domain.

Example :

Find the domain of each following function :

Function	Domain
$f(x) = x^2 + 2x - 3$	The function has no division and no even root. $D_f = \Re$ (all real numbers) $= (-\infty, \infty)$
$g(x) = \dfrac{2x}{x^2 - 9}$	The function has division. Exclude any value that make division by zero. $x^2 - 9 \neq 0$ $x \neq \pm 3$ The domain of g is : $D_g = \{$all real numbers except 3 and - 3$\}$ $= \{x \mid x \neq \pm 3\} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
$h(x) = \sqrt{3x + 12}$	The function has even root. Make sure the radicand must be positive number. $3x + 12 \geq 0 \Rightarrow x \geq -4$ The domain of h is : $D_h = \{x \mid x \geq -4\} = [-4, \infty)$

Example : Find the domain of the following function :

(a) $f(x) = \frac{2}{x+5}$ (b) $g(x) = \sqrt{x+2}$ (c) $h(x) = \frac{12x}{x^2 - 36}$

(d) $k(x) = \frac{1}{\sqrt{x-3}}$

Solution :

(a) $D_f = \{x \mid x + 5 \neq 0\} = \{x \mid x \neq -5\} = (-\infty, -5) \cup (-5, \infty)$

(b) $D_g = \{x \mid x + 2 \geq 0\} = \{x \mid x \geq -2\} = [-2, \infty)$

(c) The division : $x^2 - 36 \neq 0 \Rightarrow x \neq \pm 6$

$D_h = \{x \mid x \neq \pm 6\} = (-\infty, -6) \cup (-6, 6) \cup (6, \infty)$

(d) Since $\sqrt{x-3}$ becomes a denominator , exclude any value that make division by zero.

$x - 3 > 0 \Rightarrow x > 3$ (to avoid division by zero)

The domain : $D_k = \{x \mid x > 3\} = (3, \infty)$

2.4) Graphs of functions

Graph of a function

- The graph of a function f(x) is a graph of equation $y = f(x)$ for all x in the domain.

Graphing a function by plotting points

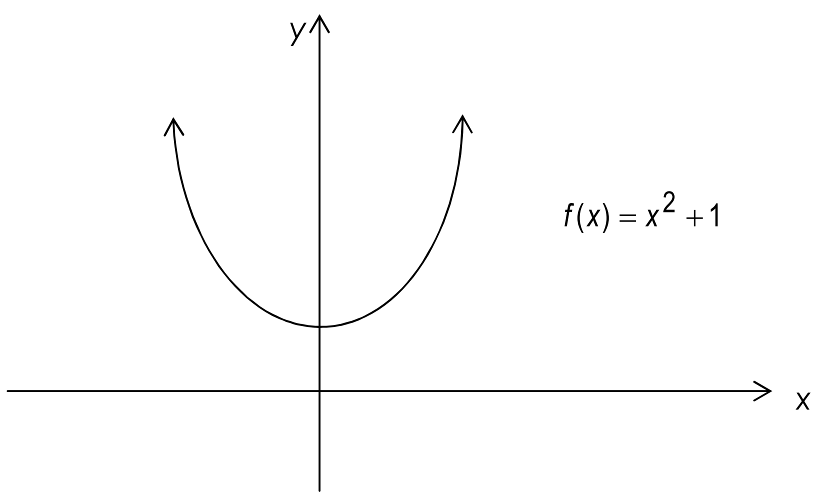
Example : Sketch the graph of $f(x) = x^2 + 1$

Solution :

1) Set up a table of values :

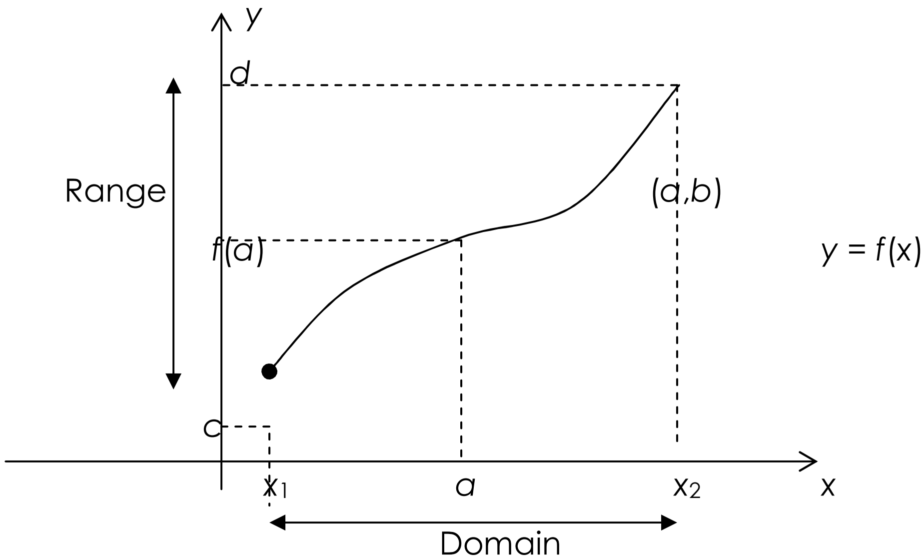
x	$y = f(x) = x^2 + 1$
-2	5
-1	2
0	1
1	2
2	5

2) Plot each ordered pair (x, y) and connect all points with a smooth curve.



Analyzing the graph of a function

Let :



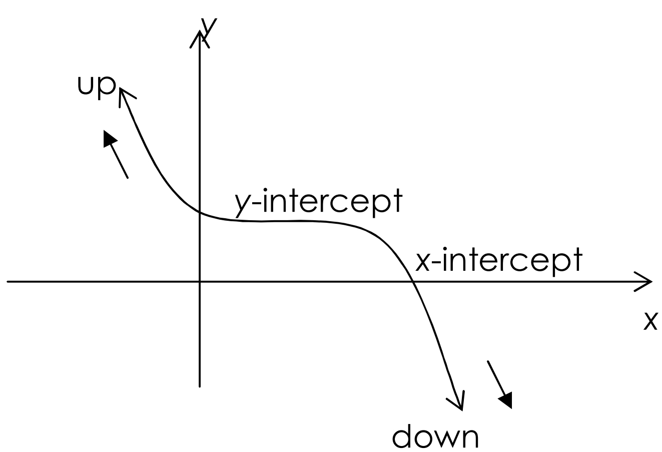
Define :

function value at $a = f(a) = b$

Domain = $[x_1 , x_2)$, Range = $[c, d)$

Open dot = the endpoint is not included .

Closed dot = the endpoint is included.



- An **arrow** means the graph extends indefinitely to the direction in which arrow points.
- **Intercept** : the points at which the graph crosses the axes. (The function can have more than one x-intercept but at most only one *y*-intercept).

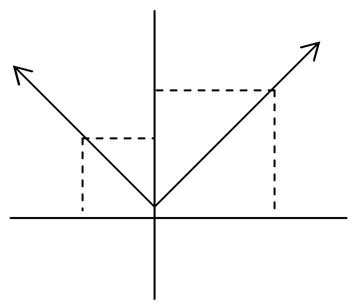
Example : Use the graph f(x) below to find :

a) domain & range

b) indicated value

c) intercept (if any)

i)



a) Domain : ________

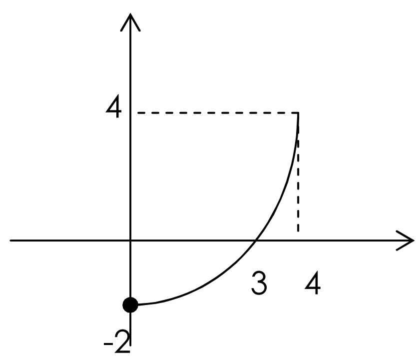
Range : __________

b) $f(-2)$ = ?

what is the value of x if $f(x) = 3$?

c) Intercept : _________________

ii)



a) Domain : ________

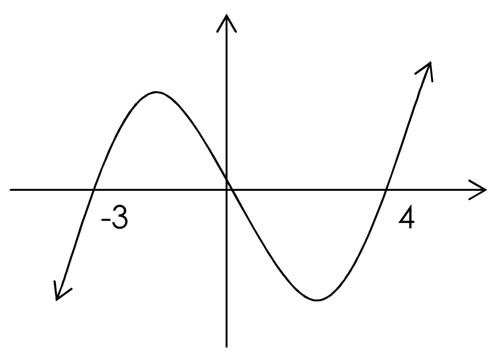
Range : __________

b) $f(3)$ = ?

what is the value of x if $f(x) = 0$?

c) Intercept : _________________

iii)



a) Domain : ________

Range : __________

b) $f(-3)$ = ?

what is the value of x if $f(x) = 0$?

c) Intercept : _________________

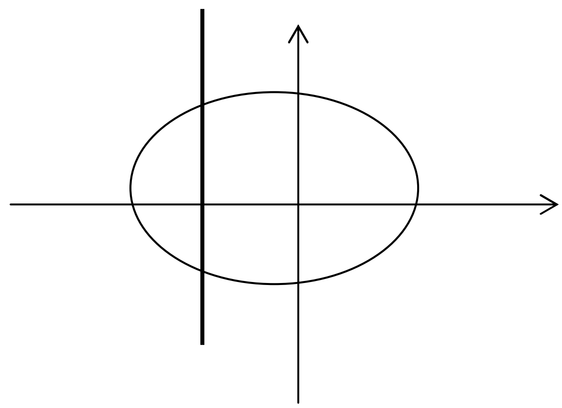
VERTICAL LINE TEST

Rule : If any vertical line **intersects** a graph in **more than one point** then the graph is **not a function**.

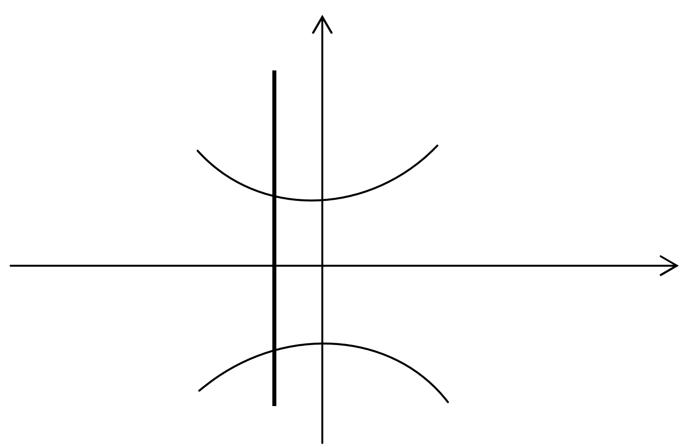
Example:

Determine whether the graphs below is a function by using vertical line test.

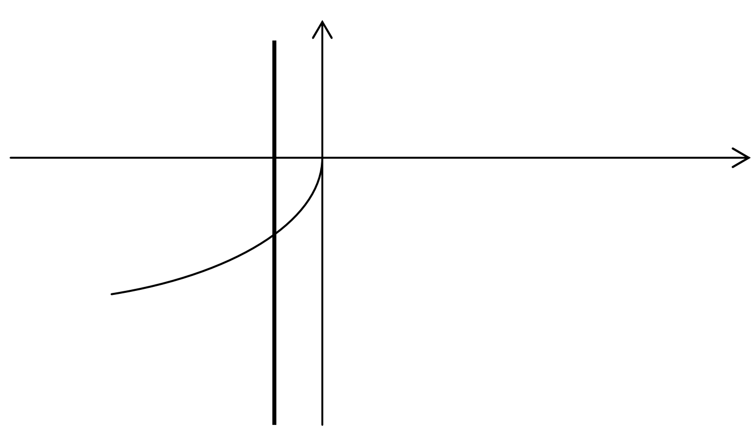
a)



b)



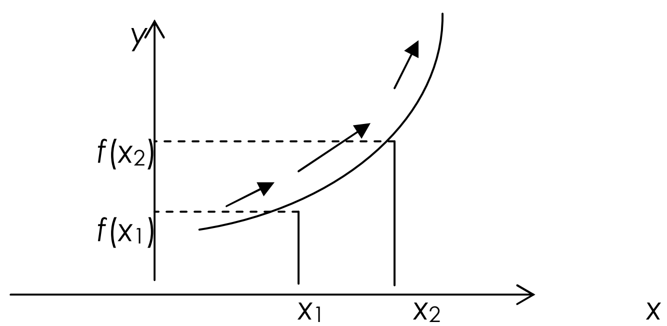
c)



Increasing and decreasing functions

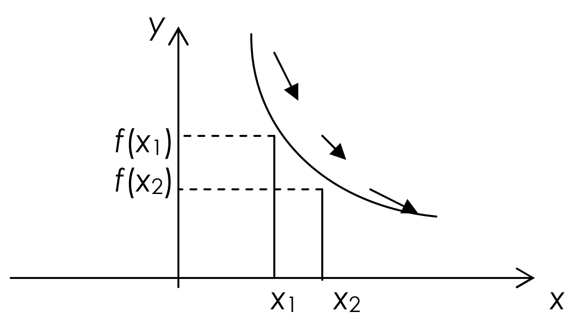
(1) A function is **increasing** on open interval I if for any x_1 , x_2 in I and $x_1 < x_2$ then :

$$f(x_1) < f(x_2)$$

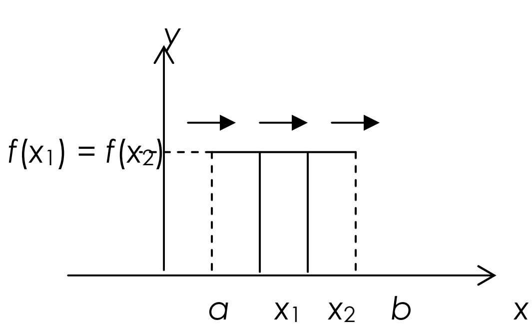


(2) A function is **decreasing** on open interval I if for any x_1 , x_2 in I and $x_1 < x_2$ then :

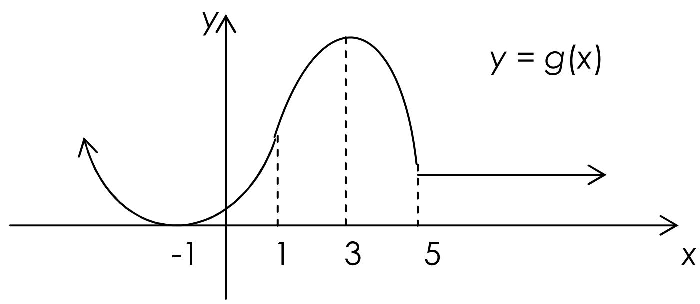
$$f(x_1) > f(x_2)$$



(3) A function is **constant** on open interval I if for any x_1 , x_2 in I and $x_1 < x_2$ then : $f(x_1) = f(x_2)$



Example :



Based on the graph, find the intervals on which a function g is increasing, decreasing or constant.

i) increasing :

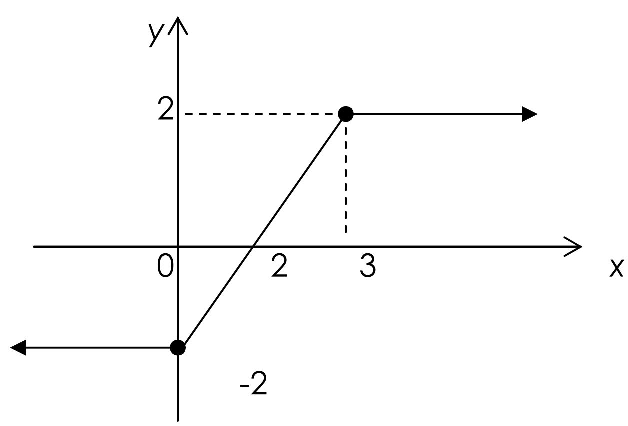
ii) decreasing :

iii) constant :

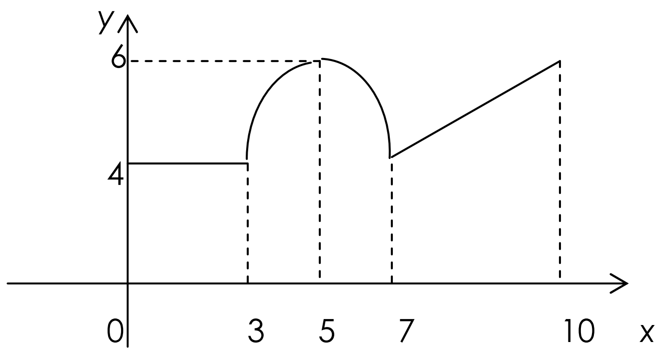
Example :

Determine the intervals on which the graph of the function below is increasing, decreasing or constant.

a)



b)



Relative Maxima and Relative Minima

Definition:

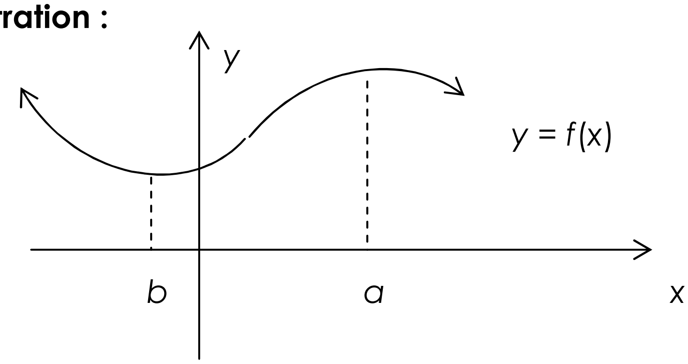
(1) A function value $f(a)$ is a **relative maximum** of f if there exists an open interval about a such that

$$f(a) > f(x)\ ,\ \text{for all } x \text{ in the interval}$$

(2) A function value $f(a)$ is a **relative minimum** of f if there exists an open interval about b such that

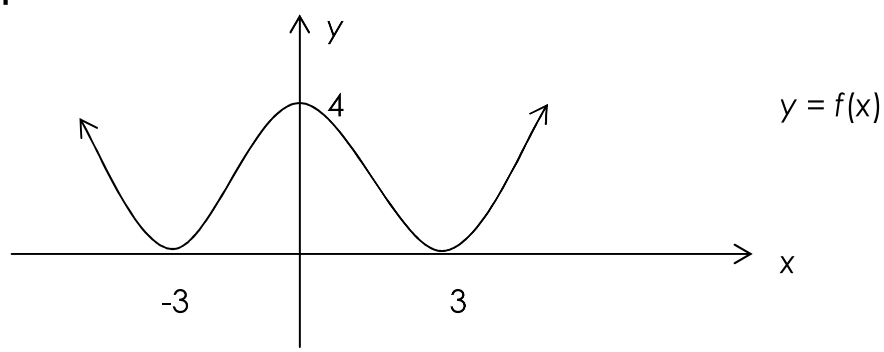
$$f(b) > f(x)\ ,\ \text{for all } x \text{ in the interval}$$

Illustration :



Relative min = $f(b)$, Relative max = $f(a)$

Example :



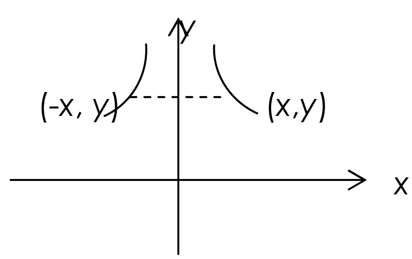
Find : a) Relative max ? b) Relative min ?

c) the numbers (if any) at which a function *f* has a relative max ?

d) the numbers (if any), at which a function *f* has a relative min ?

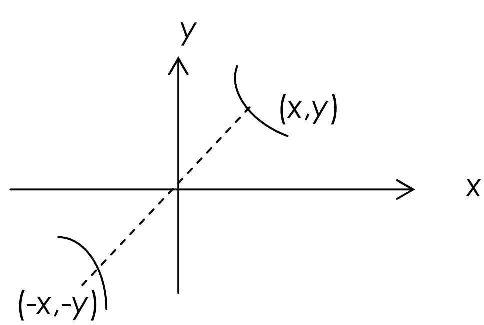
Even and Odd Functions and Symmetrical of Graph

1) If *f*(-x) = *f*(x) for all x in the domain then *f* is an **even** function



The graph of a function *f* is symmetry with respect to *y*-axis.

2) If *f*(-x) = -*f*(x) for all *x* in the domain then *f* is an **odd** function



The graph of a function f is symmetry to the origin.

Note : If the graph is not symmetry to the origin and also to y-axis then the graph f is neither function.

Example :

Determine whether the functions below are even, odd or neither function. Discuss the symmetrical of the graph of the functions.

a) $f(x) = x^3$ b) $g(x) = x^4 - 2x^2$ c) $h(x) = x^2 + 2x$

d) $f(x) = |x| + 3$ e) $g(x) = x^2\sqrt{1 - x^2}$

solution :

a) $f(x) = x^3$

Replace x by -x :

$f(-x) = (-x)^3 = -x^3 = -f(x)$ $\therefore$ an odd function

b) $g(x) = x^4 - 2x^2$

Replace x by -x :

$g(-x) = (-x)^4 - 2(-x)^2 = x^4 - 2x^2 = g(x)$ $\therefore$ an even function

c) $h(x) = x^2 + 2x$

Replace x by -x :

$h(-x) = (-x)^2 + 2(-x) = x^2 - 2x \neq h(x) \neq -h(x)$ $\therefore$ neither function

d) $f(x) = |x| + 3$

Replace x by -x :

$f(-x) = |-x| + 3 = |x| + 3 = f(x)$ $\therefore$ an even function

e) $g(x) = x^2\sqrt{1-x^2}$

Replace x by -x :

$g(-x) = (-x)^2\sqrt{1-(-x)^2} = x^2\sqrt{1-x^2}$ $\therefore$ an even function

Exercises:

Q1: Use the graphs below to identify

i) domain and range

ii) intercepts (if any)

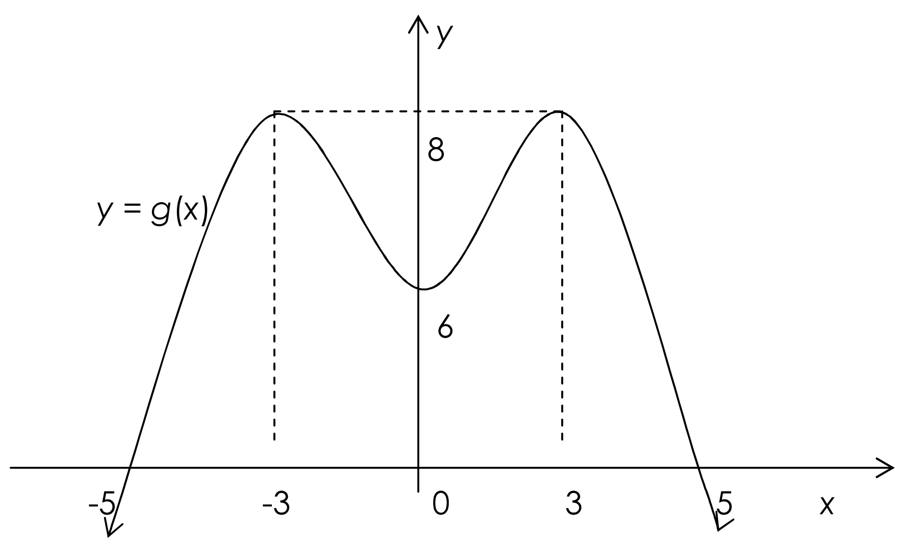
iii) symmetrical of the graph

iv) Even,odd or neither

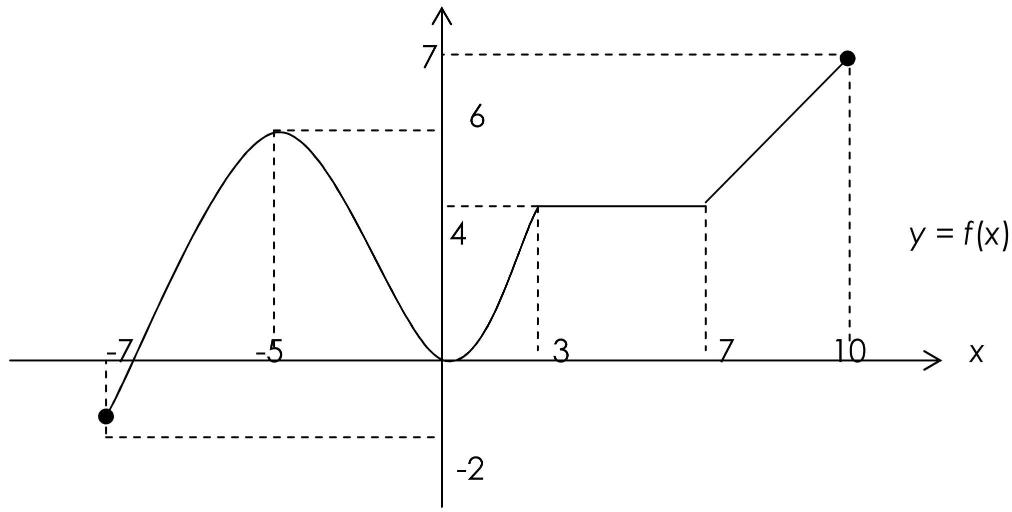
v) the intervals on which the graph is increasing, decreasing or constant

vi) Relative maximum and minimum (if any)

i)



ii)



Q2: Sketch the graph based on the informations below :

a) $f(4) = f(-4) = 0$, $f(0) = 3$, Domain = $(-\infty, \infty)$, Range = $(-\infty, 4]$

Increasing on $(-\infty, -2) \cup (0, 2)$, decreasing on $(-2, 0) \cup (2, \infty)$

Relative maxima is 4 at –2 and 2.

Relative minima is 3 at 0, f is an even function.

b) $g(0) = 2, g(1) = 0, g(3) = 0$, Domain = $(-\infty, 6)$, Range = $(-\infty, 8)$

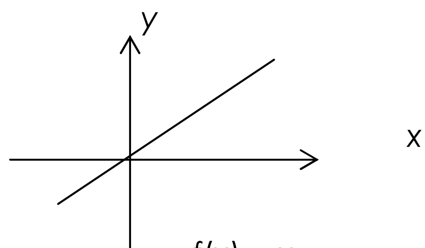
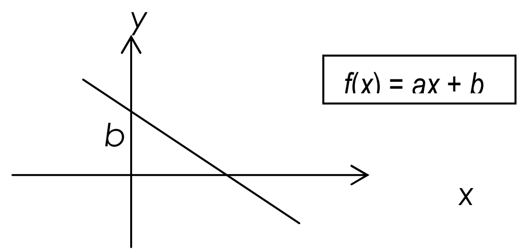
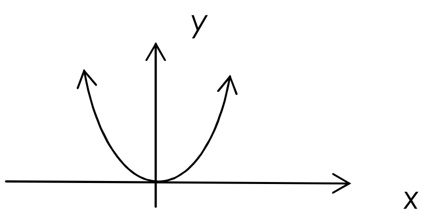
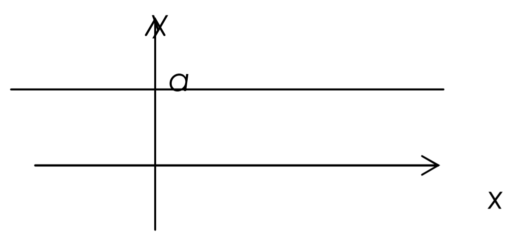
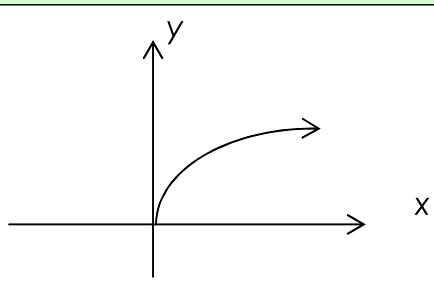
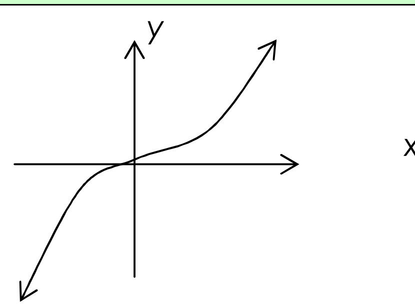
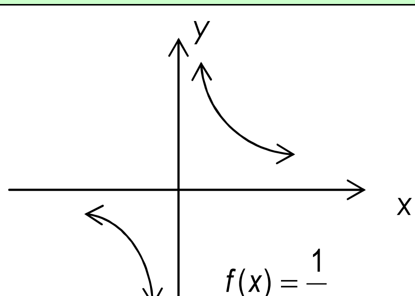
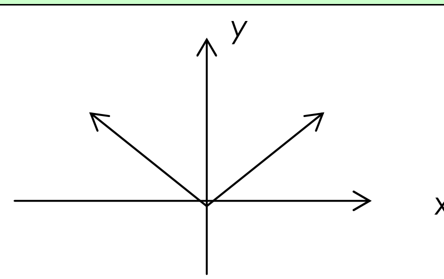
Relative maxima is 2 at 0, Relative minima is –2 at 2.

Increasing on $(-\infty, 0) \cup (2, 6)$, decreasing on (0,2)

g is neither function.

2.5) Transformation of Functions

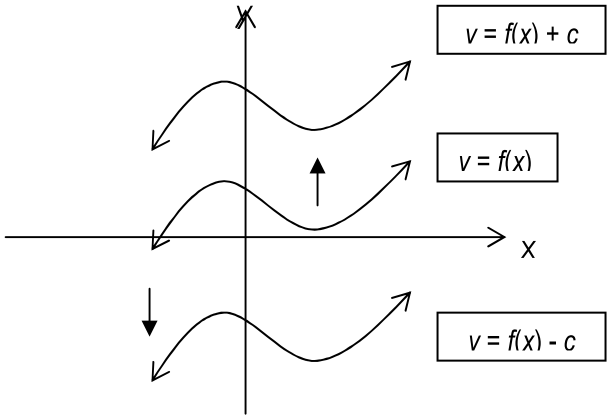
The graph of basic functions of algebra

Identity Function	Linear Function
 $f(x) = x$	
Quadratic Function	**Constant Function**
 $f(x) = x^2$	 $f(x) = a$
Square Root Function	**Cubic Function**
 $f(x) = \sqrt{x}$	 $f(x) = x^3$
Reciprocal Function	**Absolute Value Function**
 $f(x) = \frac{1}{x}$	 $f(x) = \lvert x \rvert$

TRANSFORMATION OF THE GRAPH

A) Shifting

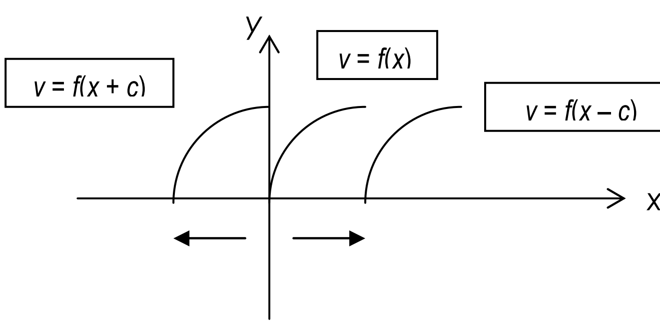
Vertical Shifting



Let $c > 0$ and *f* be a function.

- The graph of $y = f(x) + c$ is **shifting up** vertically about c units from the graph $y = f(x)$
- The graph of $y = f(x) - c$ is **shifting down** vertically about c units from the graph $y = f(x)$

Horizontal Shifting

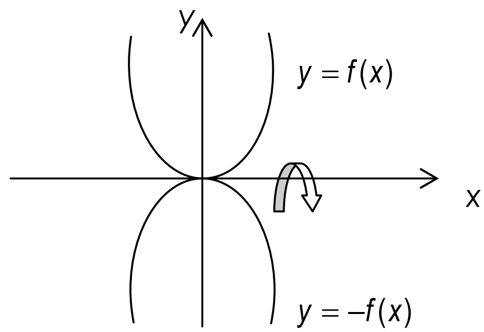


Let $c > 0$ and *f* be a function.

- The graph of $y = f(x - c)$ is **shifting to the right** horizontally about c units from the graph $y = f(x)$
- The graph of $y = f(x + c)$ is **shifting to the left** horizontally about c units from the graph $y = f(x)$

B) Reflection

The graph of $y = -f(x)$ is a reflection in the x-axis of the graph $y = f(x)$.

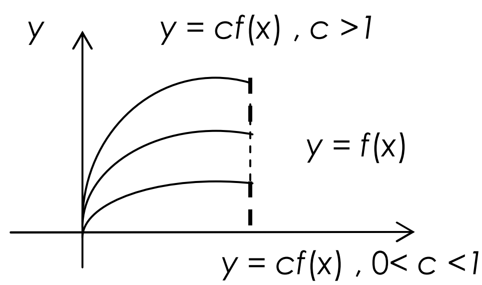


C) Stretching & Shrinking.

Let f be a function and $c > 0$ and $y = f(x)$. The graph of $y = cf(x)$ is obtained from the graph of $y = f(x)$ by multiplying each y-coordinate by c. If :

i) $c > 1$, then the graph of $y = cf(x)$ is **vertically stretching.**

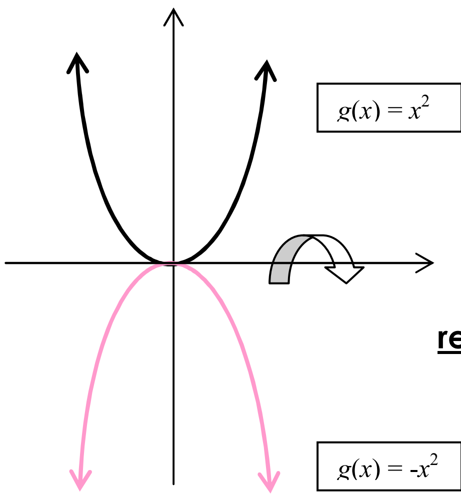
ii) $0 < c < 1$, then the graph of $y = cf(x)$ is **vertically shrinking** .



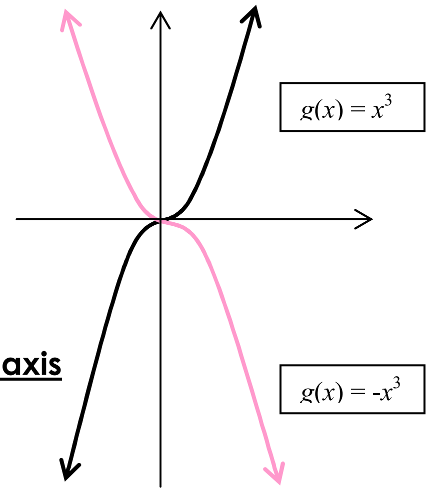
changes only for y - coordinates, no changes in x-coordinates

Examples of Transformations.

i)

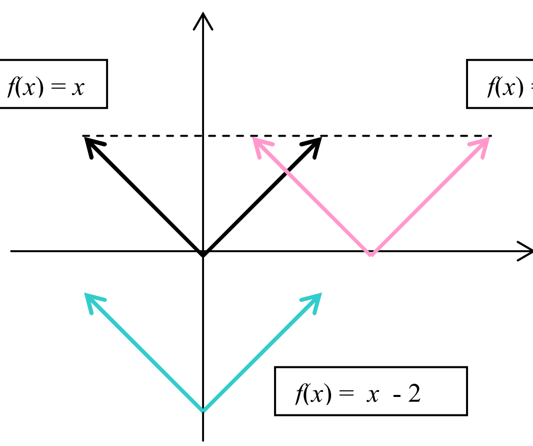


ii)

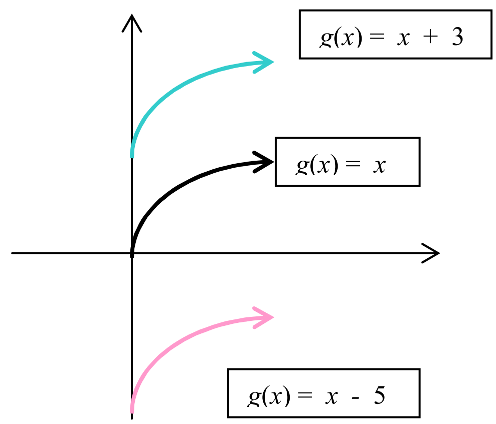


reflection to x-axis

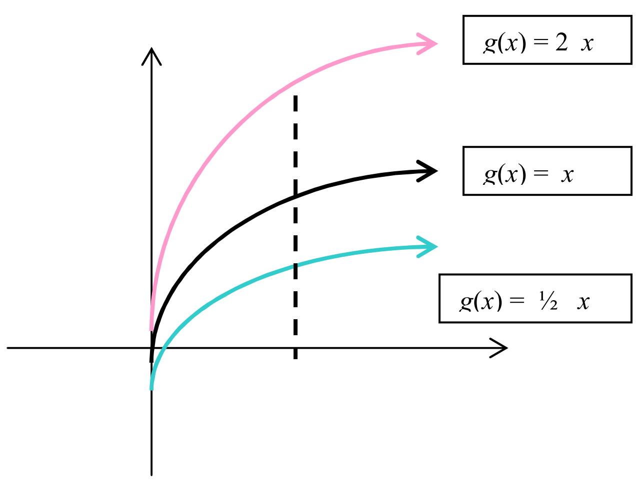
ii)



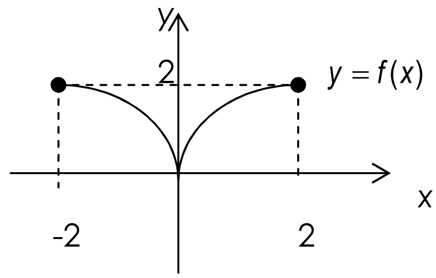
iv)



v)



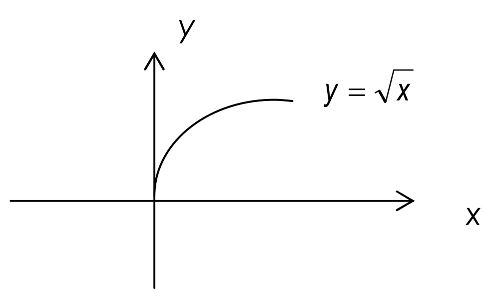
Example : Use the graph $f(x)$ below to sketch the given functions :



a) $y = f(x+2)$ b) $y = -f(x)$ c) $y = f(x) - 5$

d) $y = 2f(x)$ e) $y = \frac{1}{2}f(x)$ f) $y = -f(x+2) + 4$

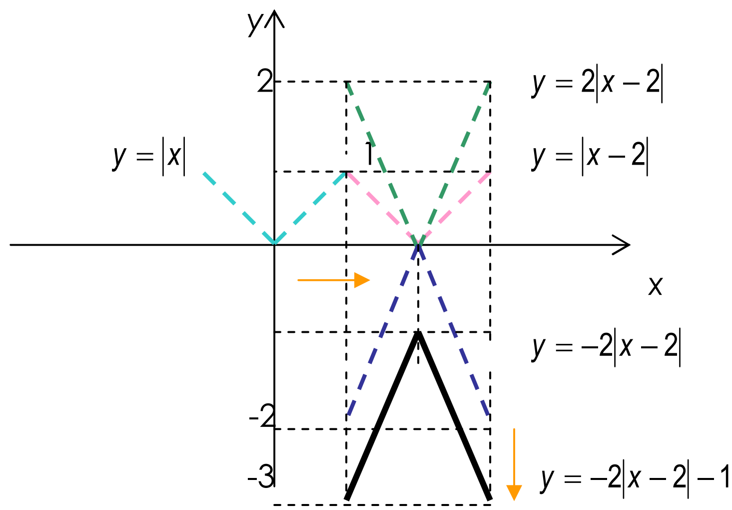
Example : Use the graph below to sketch the following functions :



a) $h(x) = \sqrt{x+1}$ b) $k(x) = -\sqrt{x} + 5$ c) $G(x) = -\sqrt{x-1} - 5$

Example : Sketch the graph of $y = -2|x-2|-1$ by using several transformations.

soln:



2.6) Combination of functions : Composite functions

The Sum, Difference, Product and Quotient of functions

Let f and g be a function

Operations	Domain
1) The **sum** of f and g $(f+g)(x) = f(x) + g(x)$	$D_{(f+g)(x)} = D_f \cap D_g$
2) The **difference** of f and g $(f-g)(x) = f(x) - g(x)$	$D_{(f-g)(x)} = D_f \cap D_g$
3) The **product** of f and g $(fg)(x) = f(x)g(x)$	$D_{(fg)(x)} = D_f \cap D_g$
4) The **quotient** of f and g $(f/g)(x) = f(x)/g(x)$ where $g(x) \neq 0$	$D_{(f/g)(x)} = D_f \cap D_g, g(x) \neq 0$

Example :

Let $f(x) = x - 5$ and $g(x) = x^2 - 1$, then find the indicated operations and its domain.

a) $(f+g)(x)$ b) $(f-g)(x)$ c) $(f\,g)(x)$ d) $(f/g)(x)$

solution :

It is known that $D_f = R, D_g = R$

a) $(f + g)(x) = f(x) + g(x) = x - 5 + x^2 - 1 = x^2 + x - 6$

$D_{(f+g)(x)} = D_f \cap D_g = R$

b) $(f - g)(x) = f(x) - g(x) = x - 5 - (x^2 - 1) = -x^2 + x - 4$

$D_{(f-g)(x)} = D_f \cap D_g = R$

c) $(f\,g)(x) = f(x)\,g(x) = (x - 5)(x^2 - 1) = x^3 - 5x - x + 5$

$D_{(fg)(x)} = D_f \cap D_g = R$

d) $(f/g)(x) = f(x) / g(x) = (x - 5)/(x^2 - 1)$

$D_{(f/g)(x)} = D_f \cap D_g$, where $g(x) \neq 0$ (the denominator cannot be zero)

$= \{x | x \in R, g(x) \neq 0\} = \{x | x \in R, x^2 - 1 \neq 0\}$ (avoid division by zero)

$= \{x | x \in R, x \neq \pm 1\}$

All real numbers except 1 and -1

Composite function

Definition :

Let *f* and *g* be a function. The composite function of *f* and *g* is defined as :

$(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$

where the domain :

$D_{(f \circ g)(x)} = \{x | x \in D_g, g \in D_f\}$

$D_{(g \circ f)(x)} = \{x | x \in D_f, f \in D_g\}$

Examples :

1) Let $f(x) = 3x - 4$ and $g(x) = x^2 + 6$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Determine the domain of each function.

solution :

$$D_f = R, D_g = R$$

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 6) = 3(x^2 + 6) - 4$$

(replacing $x^2 + 6$ for x in function *f*)

$$D_{(f \circ g)(x)} = \{x | x \in D_g, g \in D_f\} = R$$

$$(g \circ f)(x) = g(f(x)) = g(3x - 4) = (3x - 4)^2 + 6$$

(replacing 3x - 4 for x in function *g*)

$$D_{(g \circ f)(x)} = \{x | x \in D_f, f \in D_g\} = R$$

2) If $f(x) = \dfrac{2}{x-1}$ and $g(x) = \dfrac{3}{x}$, find :

a) the domain of *f* and *g*.

b) $(f \circ g)(x)$ and its domain.

c) $(g \circ f)(x)$ and its domain.

solution :

a) $D_f = \{x | x \neq 1\}, D_g = \{x | x \neq 0\}$

b) $(f \circ g)(x) = f(g(x)) = f(\dfrac{3}{x}) = \dfrac{2}{\left(\dfrac{3}{x}\right) - 1} = \dfrac{2x}{3 - x}$

$$D_{(f \circ g)(x)} = \{x | x \in D_g, g \in D_f\} = \left\{x \middle| x \in D_g, \frac{3}{x} \in D_f\right\}$$

$= \left\{x \middle| x \neq 0, \dfrac{3}{x} \neq 1\right\} = \{x | x \neq 0, x \neq 3\}$ = all real numbers except 0 and 3

c) $(g \circ f)(x) = g(f(x)) = g(\dfrac{2}{x-1}) = \dfrac{3}{2/(x-1)} = \dfrac{3(x-1)}{2}$

$$D_{(g \circ f)(x)} = \{x | x \in D_f, f \in D_g\} = \left\{x \middle| x \in D_f, \frac{2}{x-1} \in D_g\right\}$$

$= \left\{x \middle| x \neq 1, \dfrac{2}{x-1} \neq 0\right\} = \{x | x \neq 1\}$ = all real numbers except 1

2.7) Inverse Functions.

Let f and g be a function such that :

$(f \circ g)(x) = x, \quad \forall x \in D_g$ and $(g \circ f)(x) = x, \quad \forall x \in D_f$

then ***f* and *g* are the inverses of each other**.

Notation :

The inverse of f is denoted as : f^{-1}

The inverse of g is denoted as : g^{-1}

Example :

Show that each pair of functions below are inverses of each other.

a) $f(x) = 4x$, $g(x) = x/4$

b) $f(x) = 3x + 8$, $g(x) = (x - 8)/3$

solution :

a) $(f \circ g)(x) = f(g(x)) = f(x/4) = 4(x/4) = x$

$(g \circ f)(x) = g(f(x)) = g(4x) = (4x)/4 = x$

Therefore : $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, so *f* and *g* are inverses of each other.

b) $(f \circ g)(x) = f(g(x)) = f((x-8)/3) = 3(\frac{x-8}{3}) + 8 = x - 8 + 8 = x$

$(g \circ f)(x) = g(f(x)) = g(3x+8) = ((3x+8)-8)/3 = 3x/3 = x$

Therefore : $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, so *f* and *g* are inverses of each other

Finding the inverse of a function

Definition :

Let $y = f(x)$ is a one to one function . Then $x = f^{-1}(y)$ is an inverse function of f.

Steps on finding an inverse of *f*(x) :

(1) Let $y = f(x)$

(2) Solve for x

(3) Replace x in step 2 by $f^{-1}(y)$

(4) Change variable to x.

Example : Find the inverse of $f(x) = 7x - 5$

Solution:

Let $y = f(x) \Rightarrow y = 7x - 5$

Solve for x $\Rightarrow 7x = y + 5$

$\Rightarrow x = (y + 5)/7$

Replace x by $f^{-1}(y)$:

$f^{-1}(y) = (y + 5)/7$

change variable to x :

$f^{-1}(x) = (x + 5)/7$

Example : (**Exercise in class**)

Find the inverse of the following functions :

a) $f(x) = \frac{x-5}{x+3}$ b) $g(x) = \frac{5}{x-4}$

solution :

a) Let $y = f(x) \Rightarrow y = \dfrac{x-5}{x+3}$

Solve for x $\Rightarrow y(x+3) = x-5$

$$\Rightarrow xy + 3y = x - 5$$

$$\Rightarrow x - xy = 3y + 5 \Rightarrow x(1-y) = 3y + 5$$

$$\Rightarrow x = \frac{3y+5}{1-y}$$

Replace x by f^{-1}(y) :

$$\Rightarrow f^{-1}(y) = \frac{3y+5}{1-y}$$

change variable to x :

$$\Rightarrow f^{-1}(x) = \frac{3x+5}{1-x}$$

b) Let $y = g(x) \Rightarrow y = \dfrac{5}{x-4}$

Solve for x $\Rightarrow y(x-4) = 5$

$$\Rightarrow xy - 4y = 5$$

$$\Rightarrow xy = 4y + 5$$

$$\Rightarrow x = \frac{4y+5}{y}$$

Replace x by f^{-1}(y) :

$$\Rightarrow f^{-1}(y) = \frac{4y+5}{y}$$

change variable to x :

$$\Rightarrow f^{-1}(x) = \frac{4x+5}{x}$$